THE SCIENCE OF CAUSE AND EFFECT

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OUTLINE

1. The causal revolution – from associations to intervention to counterfactuals
2. The two fundamental laws of causal inference
3. From counterfactuals to problem solving
   a) policy evaluation (ATE, ETT, …)
   b) Mediation
   c) transportability – external validity
   d) missing data
   e) [attribution, selection bias, heterogeneity]

TRADITIONAL STATISTICAL INFERENCE PARADIGM

\[ P \quad \text{Joint Distribution} \quad Q(P) \quad (\text{Aspects of } P) \]

Data \quad \text{Inference}

\text{e.g., Infer whether customers who bought product } A \text{ would also buy product } B.
\[ Q = P(B \mid A) \]
FROM ASSOCIATION TO INTERVENTION

Data \rightarrow P \rightarrow P' \text{ Joint Distribution} \rightarrow Q(P') \text{ (Aspects of } P') \rightarrow \text{Inference}

e.g., Estimate \( P'(sales) \) if we double the price.
How does \( P \) change to \( P' \)? \text{New oracle}
e.g., Estimate \( P'(cancer) \) if we ban smoking.

FROM ASSOCIATION TO COUNTERFACTUALS:

Probability and statistics deal with static relations

Data \rightarrow P \rightarrow P' \text{ Joint Distribution} \rightarrow Q(P') \text{ (Aspects of } P') \rightarrow \text{Inference}

What happens when \( P \) changes?
e.g., Estimate the probability that a customer who bought \( A \) would buy \( A \) if we were to double the price.

THE STRUCTURAL MODEL PARADIGM

Data \rightarrow \text{Joint Distribution} \rightarrow \text{Data Generating Model} \rightarrow Q(M) \text{ (Aspects of } M) \rightarrow \text{Inference}

\( M \) – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

“A painful de-crowning of a beloved oracle!”
FROM STATISTICAL TO CAUSAL ANALYSIS: THE SHARP BOUNDARY

1. Causal and associational concepts do not mix.  
   **CAUSAL**  
   Spurious correlation  
   Randomization / Intervention  
   Confounding / Effect  
   Instrumental variable  
   Ignorability / Exogeneity  
   Explanatory variables  
   **ASSOCIATIONAL**  
   Regression  
   Association / Independence  
   "Controlling for" / Conditioning  
   Odds and risk ratios  
   Collapsibility / Granger causality  
   Propensity score

2.  

3.  

4.  

FROM STATISTICAL TO CAUSAL ANALYSIS:  
3. THE MENTAL BARRIERS

1. Causal and associational concepts do not mix.  
   **CAUSAL**  
   Spurious correlation  
   Randomization / Intervention  
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   Instrumental variable  
   Ignorability / Exogeneity  
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   **ASSOCIATIONAL**  
   Regression  
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   Collapsibility / Granger causality  
   Propensity score

2. No causes in → no causes out (Cartwright, 1989)

   data  
   causal assumptions  
   ⇒ causal conclusions


4. Non-standard mathematics:  
   a) Structural equation models (Wright, 1920; Simon, 1960)  
   b) Counterfactuals (Neyman-Rubin (1936), Lewis (1973))
**THE NEW ORACLE:**
STRUCTURAL CAUSAL MODELS: THE WORLD AS A COLLECTION OF SPRINGS

Definition: A structural causal model is a 4-tuple <V, U, F, P(u)>, where
- V = {V_1, ..., V_n} are endogenous variables
- U = {U_1, ..., U_m} are background variables
- F = {f_1, ..., f_n} are functions determining V,
  v_i = f_i(v, u)
- P(u) is a distribution over U

P(u) and F induce a distribution P(v) over observable variables.

**COUNTERFACTUALS ARE EMBARRASSINGLY SIMPLE**

Definition:
Y_x(u): What Y would be had X been x.
Y_x(u) is the solution for Y in a mutilated model M_x, in which the equation for X is replaced by X = x.

The Fundamental Equation of Counterfactuals:
Y_x(u) \Delta \equiv Y_{M_x}(u)

**EFFECTS OF INTERVENTIONS ARE EMBARRASSINGLY SIMPLE**

Definition:
The effect of setting X to x, P(Y = y | do(X = x)), is equal to the probability of Y = y in a mutilated model M_x, in which the equation for X is replaced by X = x.

The Fundamental Equation of Interventions:
P(Y = y | do(X = x)) \Delta \equiv P_{M_x}(Y = y)

P(Y_x = y)
**ESTIMATING THE EFFECTS OF INTERVENTIONS WITHOUT EQUATIONS**

The Fundamental Equation of Interventions:

\[
P(Y = y \mid do(X = x)) \geq P_{M_x}(Y = y)
\]

\[P(x, y, u) = P(u)P(x \mid u)P(y \mid x, u)
\]

\[P(y, u \mid do(x)) = P(u)P(y \mid x, u) \text{ Truncated product}
\]

\[P(y \mid do(x)) = \sum_{u} P(y \mid x, u) P(u) \text{ Adjustment formula}
\]

**READING COUNTERFACTUALS FROM SEM**

Data shows: \( \alpha = 0.7, \beta = 0.5, \gamma = 0.4 \)

A student named Joe, measured \( X = 0.5, Z = 1.0, Y = 1.9 \)

Q1: What would Joe’s score be had he doubled his study time?

Answer: \( Y_{Z,2} = 0.7 \times 0.5 + 0.4 \times 2.0 + \epsilon_3 = 2.30 \)

**THE TWO FUNDAMENTAL LAWS OF CAUSAL INFEERENCE**

1. The Law of Counterfactuals (and Interventions)

\[Y_{e(u)} = Y_{M_x}(u)\]

(\(M\) generates and evaluates all counterfactuals.)

2. The Law of Conditional Independence (\(d\)-separation)

\[(X \text{ sep } Y \mid Z)_{\mathcal{G}(M)} \Rightarrow (X \perp \perp Y \mid Z)_{P(v)}\]

(Separation in the model \( \Rightarrow \) independence in the distribution.)
THE LAW OF CONDITIONAL INDEPENDENCE

<table>
<thead>
<tr>
<th>Graph (G)</th>
<th>Model (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>C = f_C(U_C)</td>
</tr>
<tr>
<td>R</td>
<td>S = f_S(C,U_S)</td>
</tr>
<tr>
<td>W</td>
<td>R = f_R(C,U_R)</td>
</tr>
<tr>
<td></td>
<td>W = f_W(S,R,U_W)</td>
</tr>
</tbody>
</table>

**Gift of the Gods**

If the U’s are independent, the observed distribution P(C,R,S,W) satisfies constraints that are:
1. independent of the f’s and of P(U),
2. readable from the graph.

**D-SEPARATION: NATURE’S LANGUAGE FOR COMMUNICATING ITS STRUCTURE**

Every missing arrow advertises an independency, conditional on a separating set.

E.g., C ⊥⊥ W | (S, R)  S ⊥⊥ R | C

**Applications:**
1. Model testing
2. Structure learning
3. Reducing "what if I do" questions to symbolic calculus
4. Answering scientific questions from the graph

WHAT IF VARIABLES ARE UNOBSERVED? EFFECT OF WARM-UP ON INJURY
(Shrier & Platt, 2008)

ATE = ✔
ETT = ✔
PNC = ✔
MATHEMATICAL RESULT #1:
(Intervention is a solved problem)

- The estimability of any expression of the form
  \[ Q = P(y_1, y_2, y_3, ..., y_m | do(x_1, x_2, ..., x_n), Z_1, Z_2, ..., Z_k). \]
  Can be determined in polynomial time, given any causal graph \( G \) with both measured and unmeasured variables.
- If \( Q \) is estimable, then its estimand can be derived in polynomial time.
- The algorithm is complete.

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   e) [attribution, selection bias, heterogeneity]

MEDIATION:
A COUNTERFACTUAL TRIUMPH

1. Why decompose effects?
2. What is the definition of direct and indirect effects?
3. What are the policy implications of direct and indirect effects?
4. When can direct and indirect effect be estimated consistently from experimental and nonexperimental data?
WHY DECOMPOSE EFFECTS?

1. To understand how Nature works
2. To comply with legal requirements
3. To predict the effects of new type of interventions:
   - Signal re-routing and mechanism deactivating,
   - rather than variable fixing

LEGAL IMPLICATIONS OF DIRECT EFFECT

Can data prove an employer guilty of hiring discrimination?

(Gender) $X$ (Qualifications) $Z$ (Hiring)

What is the direct effect of $X$ on $Y$? $E(Y|do(x_1), do(z)) - E(Y|do(x_0), do(z))$

Identification is completely solved (Tian & Shpiser, 2006)

TRADITIONAL MEDIATION ANALYSIS

A CURABLE BAD HABIT

1. To prevent $M$ from varying, control for $M$, the resulting partial regression would be the direct effect.
2. Wrong! “Controlling” does not prevent $M$ from varying.
3. Example:

   $M = X + L, Y = L$
   Fixing $M = 0$ yields $Y = L$
   independent of $X$

“The best way to discuss moderation or mediation is to set aside the entire literature on these topics and start from scratch.” (Rod McDonald, 2001)
NATURAL INTERPRETATION OF AVERAGE DIRECT EFFECTS

Robins and Greenland (1992), Pearl (2001)

\[ z = f(x, u) \]
\[ y = g(x, z, u) \]

Natural Direct Effect of \( X \) on \( Y \): \( DE(x_0, x_1; Y) \)
The expected change in \( Y \) when we change \( X \) from \( x_0 \) to \( x_1 \) and, for each \( u \), we keep \( Z \) constant at whatever value it attained before the change.

\[ E[Y_{x_1}Z_{x_0} - Y_{x_0}] \]

In linear models, \( DE = \text{Controlled Direct Effect} = \beta(x_1 - x_0) \)

DEFINITION OF INDIRECT EFFECTS

\[ z = f(x, u) \]
\[ y = g(x, z, u) \]

Indirect Effect of \( X \) on \( Y \): \( IE(x_0, x_1; Y) \)
The expected change in \( Y \) when we keep \( X \) constant, say at \( x_0 \), and let \( Z \) change to whatever value it would have attained had \( X \) changed to \( x_1 \).

\[ E[Y_{x_0}Z_{x_1} - Y_{x_0}] \]

In linear models, \( IE = TE - DE \)

POLICY IMPLICATIONS OF INDIRECT EFFECTS

What is the indirect effect of \( X \) on \( Y \)?
The effect of Gender on Hiring if sex discrimination is eliminated.

GENDER \( \rightarrow \) Y \( \rightarrow \) QUALIFICATION

IGNORE \( \rightarrow \) Y \( \rightarrow \) HIRING

Deactivating a link – a new type of intervention
THE MEDIATION FORMULAS IN UNCONFOUNDED MODELS

\[ D = \mathbb{E}(Y \mid x, z) - \mathbb{E}(Y \mid x_0, z) \]

\[ E = \mathbb{E}(Z \mid x_0) \sum \]

\[ T = \mathbb{E}(Y \mid x_1) - \mathbb{E}(Y \mid x_0) \]

\[ I = \frac{\mathbb{E}(Y \mid x_1, z) - \mathbb{E}(Y \mid x_0, z)}{\mathbb{P}(Z \mid x_0)} \]

**WHAT CAN MEDIATION FORMULA DO FOR PARAMETRIC ANALYSTS?**

Multi-mediators non-linear models

\[ y = \beta_1m + \beta_2x + \beta_3m + \beta_4w + u \]

\[ m = \gamma_1x + \gamma_2w + u_2 \]

\[ w = \alpha x + u_3 \]

What combination of parameters gives the effect mediated by \( M \)?

\[ IE(M) = \beta_1(\gamma_1 + \alpha \gamma_2) \]

What combination of parameters gives the effect owed to \( M \)?

\[ TE - DE(M) = (\beta_1 + \beta_3)(\gamma_1 + \alpha \gamma_2) \]

**WHEN CAN WE IDENTIFY MEDIATED EFFECTS?**

(a) (b) (c) (d) (e) (f)

\[ (a) \quad (b) \quad (c) \quad (d) \quad (e) \quad (f) \]
WHEN CAN WE IDENTIFY MEDIATED EFFECTS?

MATHMATICAL RESULT #2: (Natural mediation is a solved problem)

• **Ignorability** is not required for identifying natural effects

• The nonparametric estimability of natural (and controlled) direct and indirect effects can be determined mechanically given any causal graph $G$ with both measured and unmeasured variables.

• If NDE (or NIE) is estimable, then its **estimand** can be derived mechanically in polynomial time.

• The algorithm is **complete** and was extended to any path-specific effect by Shpitser (2013).

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   c) **transportability** – external validity
   d) missing data
   e) [attribution, selection bias, heterogeneity]
A Theory of Causal Transportability

When can causal relations learned from experiments be transferred to another environment, different from the first, in which no experiment can be conducted.

External Validity – Decades of Literature
Cox (1958)
Campbell and Stanley (1963)
Manski (2007)

MOVING FROM THE LAB TO THE REAL WORLD . . .

Everything is assumed to be the same, trivially transportable!

Everything is assumed to be different, not transportable!

RESULT: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE

INPUT: Annotated Causal Graph
S Factors creating differences

OUTPUT:
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula
5. Completeness (Bareinboim, 2012)

\[ P^*(y\mid do(x)) = \sum_z P(y\mid do(x), z)\sum_w P^*(z\mid w)\sum_t P(w\mid do(w), t)P^*(t) \]
MATHEMATICAL RESULT #3:  
(Transportability and meta-transportability are solved)

- Nonparametric transportability of experimental results from multiple environments can be decided in polynomial time, provided commonalities and differences are encoded in selection diagrams.

- When transportability is feasible, the transport formula can be derived in polynomial time, which specifies the information needed to be extracted from each environment to synthesize a consistent estimate for the target environment.

- The algorithm is complete.

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MISSING DATA: FROM A CAUSAL INFERENCE PERSPECTIVE (Mohan, Pearl & Tian 2013)

• Pervasive in every experimental science.
• Huge literature, powerful software industry, deeply entrenched culture.
• Current practices are based on statistical characterization (Rubin, 1976) of a problem that is inherently causal.
• Needed: (1) theoretical guidance, (2) performance guarantees, and (3) tests of assumptions.

Graphical Models for Inference With Missing Data

WHAT CAN CAUSAL THEORY DO FOR MISSING DATA?

Q-1. What should the world be like, for a given statistical procedure to produce the expected result?
Q-2. Can we tell from the postulated world whether any method can produce a bias-free result? How?
Q-3. Can we tell from data if the world does not work as postulated?
• To answer these questions, we need models of the world, i.e., process models.
• Statistical characterization of the problem is too crude, e.g., MCAR, MAR, MNAR.

Distribution with missing values

Graph depicting the missingness process

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z*</th>
<th>R</th>
<th>P(Z*,X,Y,R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
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<td>0.01</td>
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<tr>
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<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>m</td>
<td>1</td>
<td>0.05</td>
</tr>
</tbody>
</table>
RECOVERABILITY AND TESTABILITY

Recoverability
Given a missingness model $G$ and data $D$, when is a quantity $Q$ estimable from $D$ without bias?
Non-recoverability
Theoretical impediment to any estimation strategy
Testability
Given a model $G$, when does it have testable implications (refutable by some partially-observed data $D'$)?

What is known about Recoverability and Testability?

<table>
<thead>
<tr>
<th></th>
<th>MCAR</th>
<th>MAR</th>
<th>MNAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recoverable</td>
<td>recoverable</td>
<td>recoverable</td>
<td>uncharted</td>
</tr>
<tr>
<td>Almost testable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncharted</td>
<td></td>
<td></td>
<td>uncharted</td>
</tr>
</tbody>
</table>

IS \( P(X,Y) \) RECOVERABLE?

THE RECOVERABILITY PIE
(and what’s in it for the user)

- Recoverability in MAR and MCAR models can be achieved by model-blind estimators (e.g., MI or EM).
- In areas (M) and (S), recoverability requires model-smart estimators.
- Testability – charted over the entire terrain.
THE PECULIAR CHARACTER OF TESTABILITY IN MISSING DATA

• Not all that looks testable is testable:
  • Some testable implications of fully recovered distributions are not testable from missing data.
  Example:
  (discomfort) X Z Y (outcome)

Now X \perp Y \mid Z is testable
  • P(X, Y, Z) is recoverable, and advertises the conditional independence X \perp Y \mid Z, which is falsifiable, hence testable.
  • Yet X \perp Y \mid Z is not falsifiable by any data in which Z is partially missing.
  • Any such data, even when generated by a model in which X \perp Y \mid Z is false, may be construed as if generated by the model above, in which X \perp Y \mid Z is true.

AN IMPOSSIBILITY THEOREM FOR MISSING DATA

(a) Accident Injury
(b) Injury Treatment

• Two statistically indistinguishable models, yet P(X, Y) is recoverable in (a) and not in (b).
• No universal algorithm exists that decides recoverability (or guarantees unbiased results) without looking at the model.

A STRONGER IMPOSSIBILITY THEOREM

(a) (b)

• Two statistically indistinguishable models, P(Y) is recoverable in both, but through two different methods:
  In (a): \( P(X) = \sum_P(Y)P(X \mid Y, R_x = 0) \), while
  in (b): \( P(X) = P(X \mid R_y = 0) \)
• No universal algorithm exists that produces an unbiased estimate whenever such exists.
THE PROBLEM OF SELECTION BIAS

• Systematic exclusion of samples from the data is a major obstacle to valid causal and statistical inferences;
• In general, it cannot be removed by randomized experiments and can hardly be detected in either experimental or passive observations.

Goal: Provide methods capable of mitigating and sometimes eliminating this bias.

(Joint work with Bareinboim & Tian)

SAMPLE SELECTION IN THE LANGUAGE OF GRAPHS

Augmented graph

\[ U_x \rightarrow c_0 \rightarrow Y \]
\[ X \rightarrow \beta_1 \rightarrow S \]
\[ Y \leftarrow \beta_2 \leftarrow S \]
\[ S = 1: \text{Included in the sample} \]
\[ S = 0: \text{Excluded from the sample} \]

TWO SOURCES OF SELECTION BIAS:

\[ X \rightarrow S \leftarrow Y \quad \text{Collider} \]
\[ X \rightarrow Y \leftarrow U_x \quad \text{Virtual collider} \]

• Cannot be eliminated by adjustment or by randomization
CONFOUNDING BIAS vs SELECTION BIAS

- Unblockable “flow” of information between treatment and outcome — spurious correlation.

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THE GENERAL SELECTION BIAS PROBLEM

**Input:** An augmented causal graph, describing a hypothesized selection process

- Under what conditions can we estimate the query (e.g., \(P(y \mid x)\)) from \(P(v \mid S = 1)\).

---

THE SELECTION BIAS PROBLEM

- Selection bias, caused by preferential exclusion of samples from the data, is a major obstacle to valid causal and statistical inferences;

- Under what conditions can we estimate the query (e.g., \(P(y \mid x)\)) from \(P(v \mid S = 1)\).
RECOVERING FROM SELECTION BIAS

**Question:** Under what conditions can we estimate the distribution \( P(y | x) \) from \( P(v | S = 1) \)?

**Theorem:** \( Q = P(y | x) \) is recoverable from selection biased data if and only if \( (S \not\perp Y | X) \).

\[
P(y | x) \text{ is not recoverable.} \quad P(y | x) \text{ is recoverable.}
\]

---

SELECTION WITH EXTERNAL INFORMATION

Estimate \( Q = P(y | x) \) from selection biased data

\[
Q \text{ is not recoverable by the previous theorem... but what if } P(W_1, W_2) \text{ is available?}
\]

---

RECOVERABILITY WITH EXTERNAL INFORMATION

**Theorem.** \( P(y | x) \) is recoverable if there is a set \( C \) such that \( (Y \perp S | C, X) \) holds in \( G \) and \( P(C, X) \) is estimable.

Moreover, \( P(y | x) = \sum_c P(y | x, c, S = 1) P(c | x) \)

\[
C = \{W_1, W_2\} \text{?yes} \\
= \{W_2, Z_1, Z_2\} \text{?no} \\
= \{W_2, Z_3\} \text{?yes}
\]
SUMMARY OF SELECTION BIAS RESULTS

- Nonparametric recoverability from selection bias can be decided provided that an augmented causal graph is available.
- When recoverability is feasible, the estimand can be derived in polynomial time.
- The result is complete for pure recoverability and sufficient for recoverability with external information.
- The back-door criterion can be generalized to handle selection bias.
- Stronger results can be obtained for the OR recoverability.

CONCLUSIONS

1. Think nature, not data, not even experiment.
2. Think hard, but only once – the rest is mechanizable.
3. Speak a language in which the veracity of each assumption can be judged by users, and which tells you whether any of those assumptions can be refuted by data.
4. Proceed in a language in which your research question can be answered from the assumptions plus the data.

Thank you
TRANSPORTABILITY OF KNOWLEDGE ACROSS DOMAINS
(with E. Bareinboim)

1. A Theory of causal transportability
   When can causal relations learned from experiments be transferred to a different environment in which no experiment can be conducted?

2. A Theory of statistical transportability
   When can statistical information learned in one domain be transferred to a different domain in which
   a. only a subset of variables can be observed? Or,
   b. only a few samples are available?

MOVING FROM THE LAB TO THE REAL WORLD...

Real world

Everything is assumed to be the same, trivially transportable!

Lab

Everything is assumed to be the different, not transportable!

MOTIVATION
WHAT CAN EXPERIMENTS IN LA TELL ABOUT NYC?

Experimental study in LA
   Measured: $P(x, y, z)$
   $P(y\,\text{do}(x), z)$
   Needed: $P^*(y\,\text{do}(x)) = \sum_z P(y\,\text{do}(x), z)P^*(z)$
   Transport Formula (calibration): $F(P, P_{\text{do}}, P^*)$
TRANSPORT FORMULAS DEPEND ON THE STORY

(a) $Z$ represents age
$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z)$

(b) $Z$ represents language skill
$P^*(y|do(x)) = P(y|do(x))$

(c) $Z$ represents a bio-marker
$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z|x)$

GOAL: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE

INPUT: Annotated Causal Graph
$S$ $\rightarrow$ Factors creating differences

OUTPUT:
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula

$P^*(y|do(x)) = \int[P(y,v,z,w,t,u|do(x));P^*(y,v,z,w,t,u)]$
TRANSPORTABILITY REDUCED TO CALCULUS

Theorem
A causal relation $R$ is transportable from $\prod$ to $\prod^*$ if and only if it is reducible, using the rules of $\textit{do}$-calculus, to an expression in which $S$ is separated from $\textit{do}(\cdot)$.

$$R(\prod^*) = P^*(y \mid \textit{do}(x)) = P(y \mid \textit{do}(x), s) = \sum_w P(y \mid \textit{do}(x), s, w) P(w \mid \textit{do}(x), s) = \sum_w P(y \mid \textit{do}(x), s) P(w \mid s) = \sum_w P(y \mid \textit{do}(x), s) P^*(w) = \sum_z R^*(z \mid w) \sum_w P(w \mid \textit{do}(w), t) P^*(t)$$

RESULT: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE

INPUT: Annotated Causal Graph
$S$ -> Factors creating differences

OUTPUT:
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula
5. Completeness (Bareinboim, 2012)

FROM META-ANALYSIS TO META-SYNTHESIS

The problem
How to combine results of several experimental and observational studies, each conducted on a different population and under a different set of conditions, so as to construct an aggregate measure of effect size that is “better” than any one study in isolation.
META-SYNTHESIS AT WORK

Target population, $[\mathcal{P}]^*$.

$X = P^*(y | do(x))$

(a) $z$

(b) $x$

(c) $y$

Meta-synthesis reduced to calculus

Theorem

\{[\mathcal{P}_1], [\mathcal{P}_2], \ldots, [\mathcal{P}_K]\} – a set of studies.

\{D_1, D_2, \ldots, D_K\} – selection diagrams (relative to $[\mathcal{P}]^*$).

A relation $R([\mathcal{P}]^*)$ is "meta estimable" if it can be decomposed into terms of the form:

$Q_k = P(V_k | do(W_k), Z_k)$

such that each $Q_k$ is transportable from $D_k$.

MATHEMATICAL RESULT #3:

(Transportability and meta-transportability are solved)

- Nonparametric transportability of experimental results from multiple environments can be decided in polynomial time, provided commonalities and differences are encoded in selection diagrams.

- When transportability is feasible, the transport formula can be derived in polynomial time, which specifies the information needed to be extracted from each environment to synthesize a consistent estimate for the target environment.

- The algorithm is complete.
DETERMINING CAUSES OF EFFECTS
A COUNTERFACTUAL VICTORY

• Your Honor! My client (Mr. A) died BECAUSE he used that drug.

• Court to decide if it is MORE PROBABLE THAN NOT that A would be alive BUT FOR the drug!

\[ P^N = P(Y|A \text{ is dead, took the drug}) \geq 0.50 \]

THE ATTRIBUTION PROBLEM

Definition:
1. What is the meaning of \( P_N(x,y) \):
   “Probability that event \( y \) would not have occurred if it were not for event \( x \), given that \( x \) and \( y \) did in fact occur.”

Answer:
\[ P_N(x,y) = P(Y_x' = y'|x,y) \]
Computable from \( M \)

Identification:
2. Under what condition can \( P_N(x,y) \) be learned from statistical data, i.e., observational, experimental and combined.
ATTRIBUTION MATHEMATIZED
(Tian and Pearl, 2000)

- Bounds given combined nonexperimental and experimental data $(P(y|x), P(y|x'))$, for all $y$ and $x$

$$\max \left\{ \begin{array}{c} 0 \\ \frac{P(y) - P(y|x')}{P(x,y)} \end{array} \right\} \leq PN \leq \min \left\{ \begin{array}{c} 1 \\ \frac{P(y|x')}{P(x,y)} \end{array} \right\}$$

- Identifiability under monotonicity (Combined data)

$$PN = \frac{P(y|x) - P(y|x')}{P(y|x)} + \frac{P(y|x') - P(y|x)}{P(x,y)}$$

CAN FREQUENCY DATA DECIDE LEGAL RESPONSIBILITY?

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Nonexperimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deaths (y)</td>
<td>16, 14</td>
</tr>
<tr>
<td>Survivals (y')</td>
<td>964, 986</td>
</tr>
<tr>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

- Nonexperimental data: drug usage predicts longer life
- Experimental data: drug has negligible effect on survival
- Plaintiff: Mr. A is special.
  1. He actually died
  2. He used the drug by choice
- Court to decide (given both data):
  Is it more probable than not that A would be alive but for the drug?

$$PN \triangleq P(Y_1 = y' \mid x, y) > 0.50$$

SOLUTION TO THE ATTRIBUTION PROBLEM

- WITH PROBABILITY ONE, $1 \leq P(y|x) \leq 1$
- Combined data tell more that each study alone