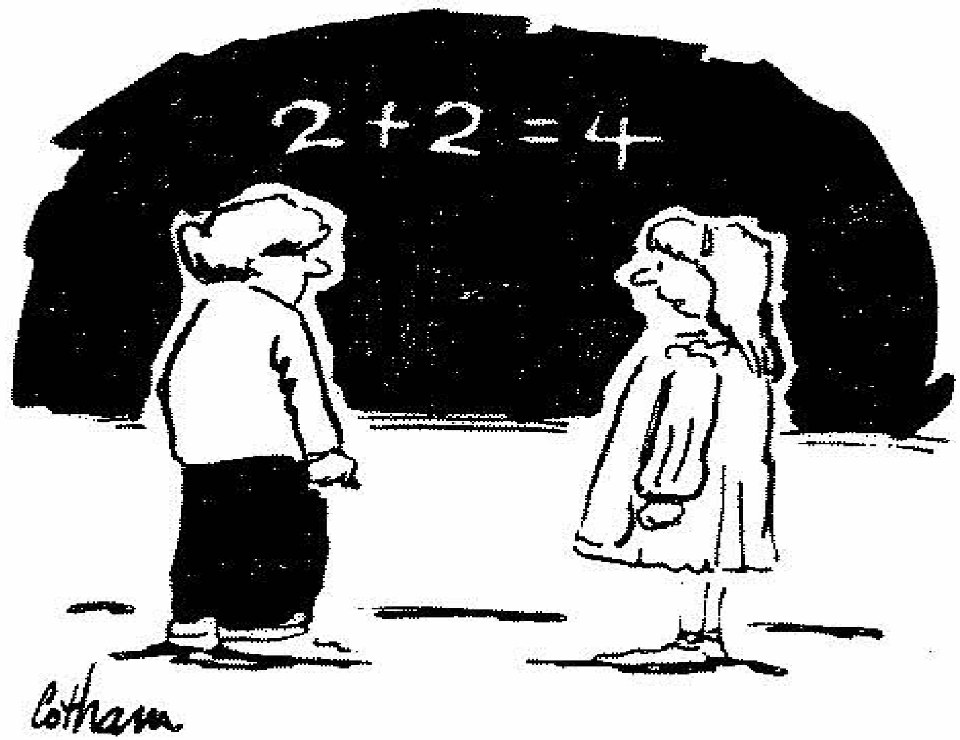
# Multilevel Model Specification Tests Using the Generalized Methods of Moment (GMM) Estimation Techniques




*"Everything gets much more complicated after this."*

Jee-Seon Kim, Edward W. Frees, Chris M. Swoboda
University of Wisconsin at Madison

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# Outline

- ❑ Correlated effects between predictors and random effects
- ❑ Econometric treatment and its limitation in multilevel models
- ❑ Omitted variable tests
- ❑ Generalized methods of moments (GMM) estimator
- ❑ Simulation results (if time permits)
- ❑ An example with National Education Longitudinal Study of 1988 (NELS:88)
- ❑ Summary

# Correlated Effects in Multilevel Models

- Consider a "student-teacher-school" model where schools are independent units: $$\mathbf{y}_s = \mathbf{X}_s\boldsymbol{\beta} + \boldsymbol{\delta}_s$$
- If a predictor is correlated with $\delta$s, it is an endogenous variable.
- If not, it is an exogenous variable.
- Standard estimation methods for multilevel models (e.g., REML, FIML, GLS, empirical Bayes estimators) assume that all predictors are exogenous, and each of these estimators can be considered as a random effects estimator.
- The exogeneity assumption is often violated when relevant variables are omitted. Even a moderate correlation between the **X**s and $\delta$s may result in severe bias in the estimation of $\beta$ as well as variance components (i.e., omitted variable bias).
- In econometrics, correlated effects are examined by comparing a random effects estimator to a fixed effects estimator.

# Model Specification Test in Econometrics

- $b_{FE}$ is robust to misspecification due to omitted person variables.
- Uses within-person deviations.
- Does not estimate person or higher-level effects.

Ho: ***No misspecification***

| **Fixed Effects Estimator $b_{FE}$** | **Random Effects Estimator $b_{RE}$** |
|---|---|

- $b_{RE}$ assumes the most stringent form of exogeneity.
- Biased if there exist omitted person variables.
- Default and only option in most software.

Hausman (1978): Specification test for econometrics. *Econometrica*.

$$\chi^2 = \frac{(b_{FE} - b_{RE})^2}{VAR(b_{FE}) - VAR(b_{RE})}$$

If the difference is significant, reject Ho & use robust $b_{FE}$.

If not, one would prefer efficient $b_{RE}$.

# Limited Use of Econometric Treatment in ML Models

- The Hausman test is an effective tool for panel data models, (two-level random intercept models for longitudinal data), but its use is limited for more complex models.
- A level-3 FE estimator would be biased if there exist omitted variables at level 2. The Hausman test would compare two biased estimators.
- This is not a problem of the Hausman test itself, but rather is an improper application of the test for which it is not designed.

# Mean Math Scores for Subpopulations in NELS:88

| | | Listwise Deletion (N=5278) | Multiple Imputation (N=7334) |
|---|---|---|---|
| *Student* | 10th grade math score | 45.43 | 44.35 |
| | 8th grade math score | 37.85 | 36.97 |
| | Minority | 40.43 | 39.11 |
| | Caucasian | 47.01 | 46.37 |
| *Teacher* | Math Background | 46.30 | 45.51 |
| | No Math Background | 43.41 | 42.14 |
| | Female | 44.20 | 42.84 |
| | Male | 44.31 | 43.21 |
| | Minority | 38.80 | 37.26 |
| *School* | Urban | 45.36 | 43.33 |
| | Rural | 42.53 | 41.47 |
| | Public School | 43.02 | 41.48 |
| | Private School | 52.18 | 51.21 |

# Omitted Variable Tests in ML Models

- Kim and Frees (2006) provided model specification tests for general multilevel models with additional levels and random slopes.

- The method allows for FE estimators at different levels (e.g., $b_{FEteacher}$, $b_{FEschool}$) and for testing the effects of unobserved variables at different levels (e.g., $\mathbf{U}_{teacher}$, $\mathbf{U}_{school}$), jointly or separately.

- One can conduct three types of omitted variable tests.

| Omitted Variable Tests | Hypothesis | Estimators compared |
|---|---|---|
| 1. Multiple-level test | Ho: $U_{teacher}=U_{school}=0$ | $b_{FEt}$ vs. $b_{RE}$ |
| 2. Intermediate-level test | Ho: $U_{teacher}=0$ | $b_{FEt}$ vs. $b_{FEs}$ |
| 3. Highest-level test | Ho: $U_{school}=0$ | $b_{FEs}$ vs. $b_{RE}$ |

# Omitted Variable Tests in a 3-Level Model for NELS:88

| | | $b_{FEt}$ | $b_{RE}$ |
|---|---|---|---|
| *Student* | Prior Achievement | 0.85* | 0.95* |
| | SES | 0.71* | 1.11* |
| | *Female* | 0.14 | 0.17 |
| | Minority | -0.55 | -0.76* |
| *Teacher* | Math Background | | 0.53* |
| | Experienced | | 0.45 |
| | Female | | 0.39* |
| | Minority | | -0.42 |
| *School* | Urban | | -0.43 |
| | Rural | | -0.37 |
| | School Size/100 | | 0.02 |
| | % Caucasian/10 | | 0.01 |
| | % Single Parent Homes/10 | | -0.10 |
| | Public School | | -0.93* |

OV Test 1 $H_0$: Uteacher=Uschool=0. $b_{FEt}$ vs. $b_{RE}$. chi-sq=143.34, df=4, $p<0.01$.

# Omitted Variable Tests in a 3-Level Model for NELS:88

| | | $b_{FEt}$ | $b_{FEs}$ |
|---|---|---|---|
| *Student* | Prior Achievement | 0.85* | 0.94* |
| | SES | 0.71* | 0.94* |
| | *Female* | 0.14 | 0.17 |
| | Minority | -0.55 | -0.64* |
| *Teacher* | Math Background | | 0.71* |
| | Experienced | | 0.56 |
| | Female | | 0.42 |
| | Minority | | -0.36 |
| *School* | Urban | | |
| | Rural | | |
| | School Size/100 | | |
| | % Caucasian/10 | | |
| | % Single Parent Homes/10 | | |
| | Public School | | |

OV Test 1 $H_0$: Uteacher=Uschool=0. $b_{FEt}$ vs. $b_{RE}$. chi-sq=143.34, df=4, p<0.01.
OV Test 2 $H_0$: Uteacher=0. $b_{FEt}$ vs. $b_{FEs}$. chi-sq =130.64, df=4, p<0.01.

# Omitted Variable Tests in a 3-Level Model for NELS:88

| | | $b_{FEs}$ | $b_{RE}$ |
|---|---|---|---|
| *Student* | Prior Achievement | 0.94* | 0.95* |
| | SES | 0.94* | 1.11* |
| | ***Female*** | 0.17 | 0.17 |
| | Minority | -0.64* | -0.76* |
| *Teacher* | Math Background | 0.71* | 0.53* |
| | Experienced | 0.56 | 0.45 |
| | Female | 0.42 | 0.39* |
| | Minority | -0.36 | -0.42 |
| *School* | Urban | | -0.43 |
| | Rural | | -0.37 |
| | School Size/100 | | 0.02 |
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OV Test 1 $H_0$: Uteacher=Uschool=0. $b_{FEt}$ vs. $b_{RE}$. chi-sq=143.34, df=4, p<0.01.
OV Test 2 $H_0$: Uteacher=0. $b_{FEt}$ vs. $b_{FEs}$. chi-sq =130.64, df=4, p<0.01.
OV Test 3 $H_0$: Uschool=0. $b_{FEs}$ vs. $b_{RE}$.

# Omitted Variable Tests in a 3-Level Model for NELS:88

| | | Unbiased | Biased | |
|---|---|---|---|---|
| | | bFEt | bFEs | bRE |
| *Student* | Prior Achievement | 0.85* | 0.94* | 0.95* |
| | SES | 0.71* | 0.94* | 1.11* |
| | *Female* | 0.14 | 0.17 | 0.17 |
| | Minority | -0.55 | -0.64* | -0.76* |
| *Teacher* | Math Background | | 0.71* | 0.53* |
| | Experienced | | 0.56 | 0.45 |
| | Female | | 0.42 | 0.39* |
| | Minority | | -0.36 | -0.42 |
| *School* | Urban | | | -0.43 |
| | Rural | | | -0.37 |
| | School Size/100 | | | 0.02 |
| | % Caucasian/10 | | | 0.01 |
| | % Single Parent Homes/10 | | | -0.10 |
| | Public School | | | -0.93* |

OV Test 1 $H_0$: Uteacher=Uschool=0. $b_{FEt}$ vs. $b_{RE}$. chi-sq=143.34, df=4, $p<0.01$.
OV Test 2 $H_0$: Uteacher=0. $b_{FEt}$ vs. $b_{FEs}$. chi-sq =130.64, df=4, $p<0.01$.
OV Test 3 $H_0$: Uschool=0. $b_{FEs}$ vs. $b_{RE}$ is not meaningful as $b_{FEs}$ is biased.
Although $b_{FEt}$ is unbiased, it does not provide estimates at levels 2 and 3.

# Generalized Methods of Moment (GMM) Estimator

- Kim and Frees (2007) presented a GMM estimator for multilevel models.

$$\mathbf{b}_{\mathbf{GMM}} = (\mathbf{X'W}P(\mathbf{H})\mathbf{WX})^{-}\mathbf{X'W}P(\mathbf{H})\mathbf{Wy}$$

**H** is a set of instruments (def: exogenous variables used for estimating β)
$P(\mathbf{H})=\mathbf{H}(\mathbf{H}'\mathbf{H})^{-}\mathbf{H}'$ is the projection spanned by **H**. **W**: weight matrix.

- In instrumental variable (IV) methods, one uses additional variables outside of the model (i.e., instruments) when some **X** are endogenous.
- In contrast, the multilevel GMM approach can compose instruments **H** using functions of **X** without external variables.
- Thus, in this "internal" IV approach, external variables can be included in **H** but are not required.
- Also, $\mathbf{b}_{\mathbf{GMM}}$ encompasses the fixed effects (FE) and random effects (RE) estimators as special cases and provides additional options.

# FE, RE, and the GMM Estimator as a Middle Ground

- The FE estimator is obtained:
  - based on the idea that all predictors might be endogenous
  - by using only within-group deviations as instruments
- The RE estimator is obtained:
  - based on the assumption that all predictors are exogenous
  - by using both within-group deviations and group means as instruments
- The GMM estimator takes a middle-of-the-road approach:
  - regards some predictors as endogenous and the rest as exogenous
  - uses both within and between group information of exogenous variables
  - use only within-group information of endogenous variables as instruments
  - The GMM estimator can be as robust as the FE estimator and provides estimates at all levels.

# Estimator Continuum (Robust to Efficient)

***most robust*** ... ***most efficient***

| | | **bFEt** | **bGMMt** | **bFEs** | **bGMMs** | **bRE** |
|---|---|---|---|---|---|---|
| *Student* | Prior Achievement | 0.85* | | 0.94* | | 0.95* |
| | SES | 0.71* | | 0.94* | | 1.11* |
| | *Female* | 0.14 | | 0.17 | | 0.17 |
| | Minority | -0.55 | | -0.64* | | -0.76* |
| *Teacher* | Math Background | | | 0.71* | | 0.53* |
| | Experienced | | | 0.56 | | 0.45 |
| | Female | | | 0.42 | | 0.39* |
| | Minority | | | -0.36 | | -0.42 |
| *School* | Urban | | | | | -0.43 |
| | Rural | | | | | -0.37 |
| | School Size/100 | | | | | 0.02 |
| | % Caucasian/10 | | | | | 0.01 |
| | % Single Parent Homes/10 | | | | | -0.10 |
| | Public School | | | | | -0.93* |

Recall that only **bFEt** is unbiased and both **bFEs** and **bRE** are biased. Although unbiased, **bFEt** does not provide estimates at higher levels. We can obtain GMM estimators and create an estimator continuum.

# Estimator Continuum (Robust to Efficient)

*most robust* → *most efficient*

| | | bFEt | bGMMt | bFEs | bGMMs | bRE |
|---|---|---|---|---|---|---|
| *Student* | Prior Achievement | 0.85* | 0.85* | 0.94* | 0.94* | 0.95* |
| | SES | 0.71* | 0.71* | 0.94* | 0.94* | 1.11* |
| | *Female* | 0.14 | 0.12 | 0.17 | 0.17 | 0.17 |
| | Minority | -0.55 | -0.55 | -0.64* | -0.68* | -0.76* |
| *Teacher* | Math Background | | 0.58* | 0.71* | 0.52* | 0.53* |
| | Experienced | | 0.59 | 0.56 | 0.47 | 0.45 |
| | Female | | 0.39 | 0.42 | 0.40* | 0.39* |
| | Minority | | 0.16 | -0.36 | -0.27 | -0.42 |
| *School* | Urban | | 0.18 | | -0.28 | -0.43 |
| | Rural | | -0.54 | | -0.39 | -0.37 |
| | School Size/100 | | 0.08* | | 0.04 | 0.02 |
| | % Caucasian/10 | | 0.52* | | 0.12 | 0.01 |
| | % Single Parent Homes/10 | | -0.05 | | -0.08 | -0.10 |
| | Public School | | -1.94* | | -1.07* | -0.93* |

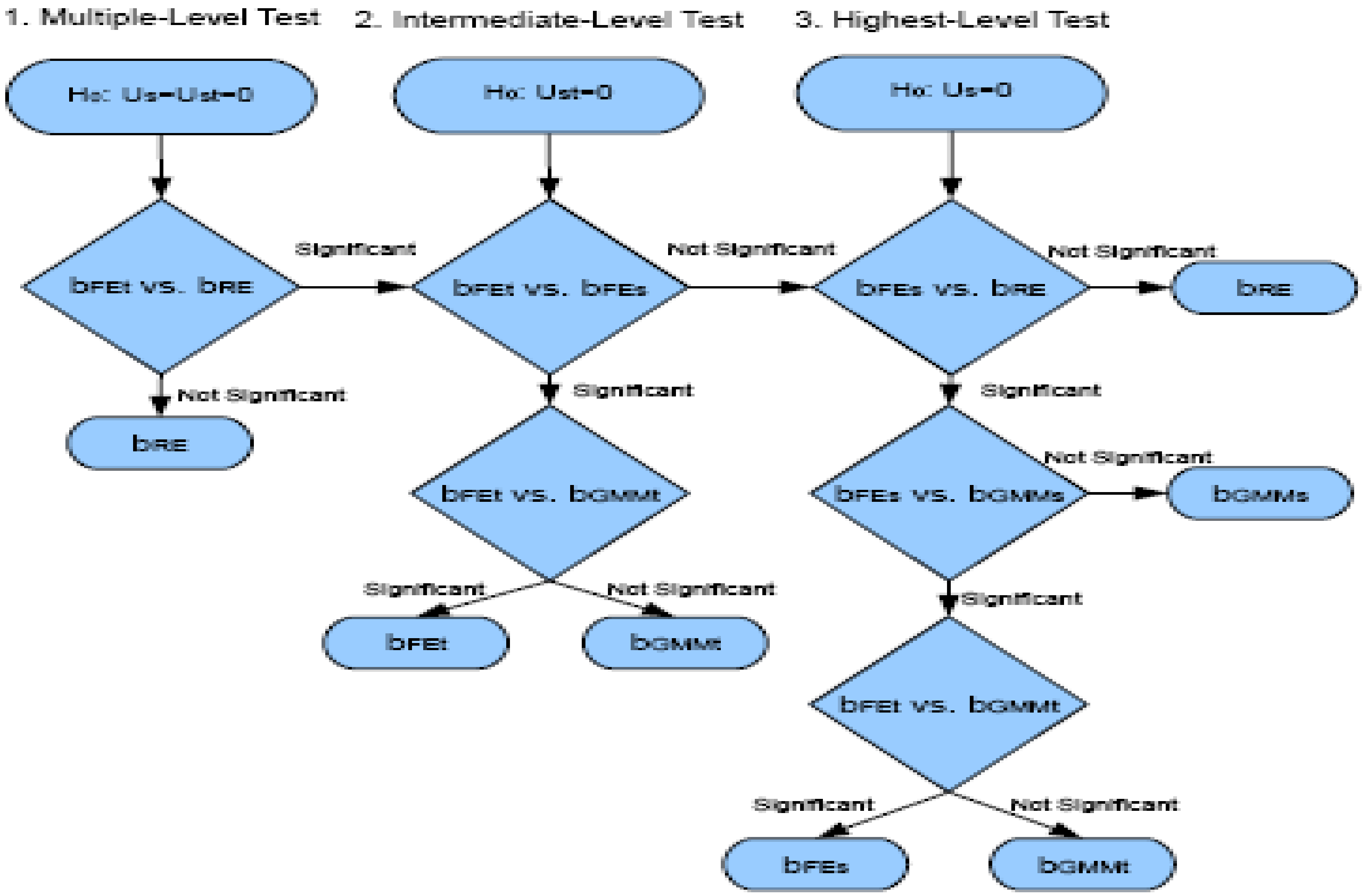
Prior achievement, SES, and % Caucasian might be endogenous.
We can obtain two GMM estimators at the teacher and school levels. Within teacher (school) deviations and teacher (school) means as instruments.

# Estimator Continuum (Robust to Efficient)

| | | Unbiased | | Biased | | |
|---|---|---|---|---|---|---|
| | | ***b*FEt** | ***b*GMMt** | ***b*FEs** | ***b*GMMs** | ***b*RE** |
| *Student* | Prior Achievement | 0.85* | 0.85* | 0.94* | 0.94* | 0.95* |
| | SES | 0.71* | 0.71* | 0.94* | 0.94* | 1.11* |
| | *Female* | 0.14 | 0.12 | 0.17 | 0.17 | 0.17 |
| | Minority | -0.55 | -0.55 | -0.64* | -0.68* | -0.76* |
| *Teacher* | Math Background | | 0.58* | 0.71* | 0.52* | 0.53* |
| | Experienced | | 0.59 | 0.56 | 0.47 | 0.45 |
| | Female | | 0.39 | 0.42 | 0.40* | 0.39* |
| | Minority | | 0.16 | -0.36 | -0.27 | -0.42 |
| *School* | Urban | | 0.18 | | -0.28 | -0.43 |
| | Rural | | -0.54 | | -0.39 | -0.37 |
| | School Size/100 | | 0.08* | | 0.04 | 0.02 |
| | % Caucasian/10 | | 0.52* | | 0.12 | 0.01 |
| | % Single Parent Homes/10 | | -0.05 | | -0.08 | -0.10 |
| | Public School | | -1.94* | | -1.07* | -0.93* |

We found two out of five estimators are unbiased for the three-level model for NELS:88. Therefore, we chose **bGMMt**, which is the most efficient among unbiased estimators.

# Guideline for Omitted Variable Tests and Optimal Estimator
## Kim & Swoboda (2008)



# Individual Coefficient Tests

| | | *b*GMMt | *b*RE | chi-sq (*df*=1) |
|---|---|---|---|---|
| *Student* | Prior Achievement | 0.85* | 0.95* | 108.70** |
| | SES | 0.71* | 1.11* | 9.85** |
| | *Female* | 0.12 | 0.17 | 0.42 |
| | Minority | -0.55 | -0.76* | 1.11 |
| *Teacher* | Math Background | 0.58* | 0.53* | 0.20 |
| | Experienced | 0.59 | 0.45 | 0.73 |
| | Female | 0.39 | 0.39* | 0.00 |
| | Minority | 0.16 | -0.42 | 3.55 |
| *School* | Urban | 0.18 | -0.43 | 5.04* |
| | Rural | -0.54 | -0.37 | 1.15 |
| | School Size/100 | 0.08* | 0.02 | 11.69** |
| | % Caucasian/10 | 0.52* | 0.01 | 14.20** |
| | % Single Parent Homes/10 | -0.05 | -0.10 | 0.44 |
| | Public School | -1.94* | -0.93* | 23.72** |

# Summary

- This study presents statistical methods for handling endogeneity in multilevel models due to omitted variables.
- In the social sciences, researchers cannot always design a study that is appropriate for addressing misspecification issues (e.g., observational or quasi-experimental studies).
- Our NELS:88 analysis reveals that under the exogeneity assumption, the random effects estimator underestimates the effects of school variables (e.g., school size, % Caucasian), while overestimating the effects of student variables (e.g., minority, SES, prior achievement).
- Thus, the unbiased GMM estimator is recommended.
- Multilevel data contain rich information, and the GMM estimation technique exploits the hierarchical structure for testing correlated effects and for obtaining alternative robust estimators when some predictors are endogenous.

# References

Kim & Frees (2006). Omitted variables in multilevel models. *Psychometrika*

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Kim & Swoboda (in press). Handling omitted variable bias. In J.J. Hox & J.K. Roberts (Eds.), *Handbook of advanced multilevel analysis*.

Email to Jee-Seon Kim at jeeseonkim@wisc.edu for reprint.