Regularized Reduced-Rank Growth Curve Models

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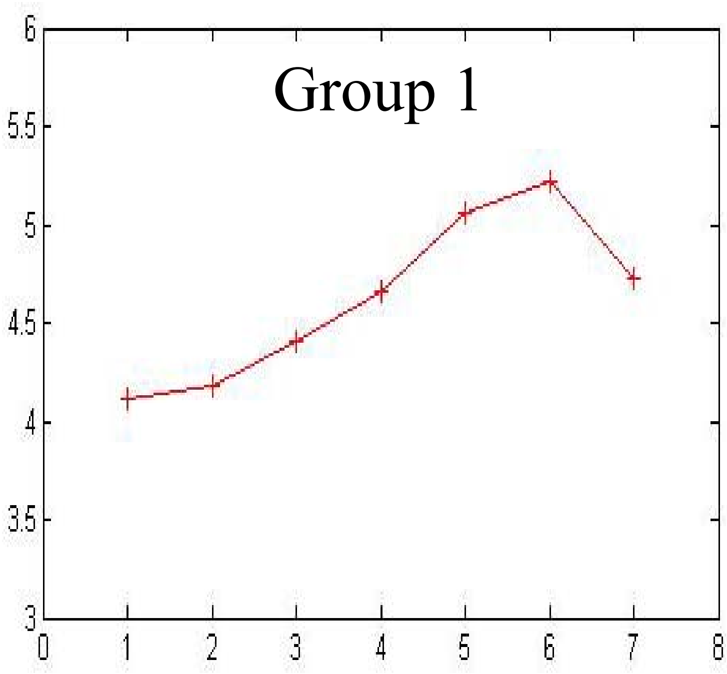
Overview

1. Growth Curve models

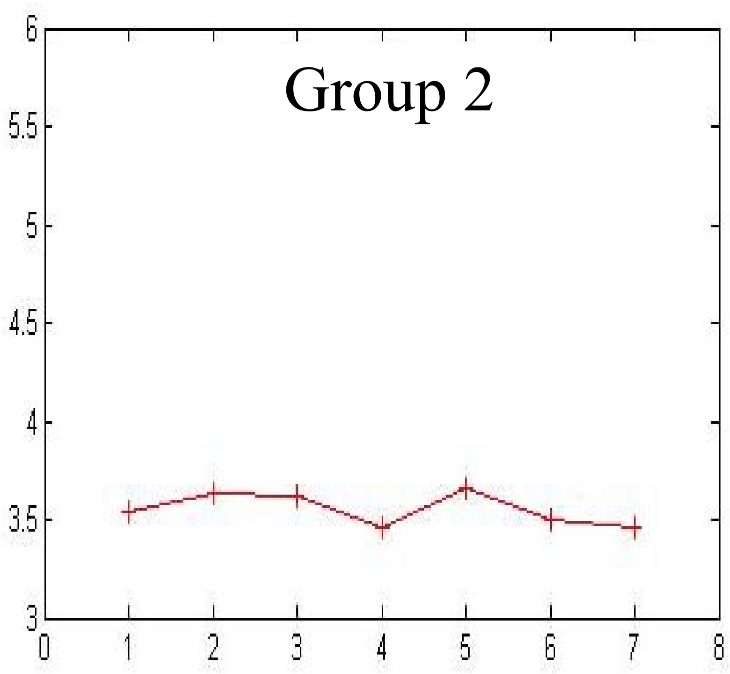
2. Reduced-Rank Growth Curve models

3. Regularized Reduced-Rank Growth Curve models

Growth Curve Models (GCMs)



7 time points



7 time points

Growth Curve Models (GCMs)

- The model for GCMs

$$Y = XBH' + E$$ (Potthoff & Roy, 1964)

- Least Squares (LS) estimate of B

$$\hat{B} = (X'X)^{-1}X'Y\,H(H'H)^{-1}$$

Reduced-Rank Growth Curve Models

- Reduced-Rank Growth Curve Models
  (Reinsel & Velu, 1998)

$\mathrm{rank}(B) = r \leq \min(q, d)$

- Reduced-rank least squares (LS) estimate of B

Step. 1

$$\mathrm{GSVD}(\hat{B})_{X'X, H'H}$$

Step. 2

$$\tilde{B} = {}_q\tilde{U}_r \, {}_r\tilde{D}_r \, {}_r\tilde{V}'_d$$

Regularized Reduced Rank GCMs

- A ridge type of regularization

Small positive values of regularization parameters (λ, ρ) typically shrink an estimate of B, closer to the true parameter values (Hoerl & Kennard, 1970)

- Small sample size
- High collinearity among predictor variables (X) and/or among trend coefficients (H)

Regularized Reduced Rank GCM

- Minimize

$$\phi_{\lambda,\rho}(B) = SS(Y - XBH') + \lambda SS(B)_{X'X,I} + \rho SS(B)_{I,H'H} + \lambda\rho SS(B)_{X'X,H'H}$$

- Regularized reduced-rank LS estimate of B

Step. 1

- Regularized unconstrained-rank LS estimate of B

$$\hat{B}(\lambda,\rho) = (X'X + \lambda I)^{-1} X'Y H(H'H + \rho I)^{-1}$$

(λ, ρ : ridge parameters, I: identity matrix)

Step. 2

- GSVD$(\hat{B}(\lambda,\rho))_{X'X+\lambda I, H'H+\rho I}$

Two important choices

1. The choice of the best dimensionality (r) of B
   - Permutation tests

2. The choice of optimal values λ and ρ
   - The K-fold cross validation method

An example

- Grizzle and Allen's data set (1969)
  - Y : level of coronary sinus potassium over 7 time points (1, 3, 5, 7, 9, 11, and 13 minutes after occlusion)
  - X : 4 experimental groups (n=36) (Unequal sample size: n1=9, n2=10, n3=8, n4=9)
  - H: Up to the third order (cubic) orthogonal polynomials

Permutation Tests Results

λ	ρ	DIM	p-value
0	0	1	0
0	0	2	0.966
0	0.1	1	0.001
0	0.1	2	0.968
0	0.2	1	0.001
0	0.2	2	0.979
0	0.5	1	0
0	0.5	2	0.971
0	1	1	0.001
0	1	2	0.985
0.2	0	1	0
0.2	0	2	0.974
0.2	0.1	1	0.001
0.2	0.1	2	0.969
0.2	0.2	1	0
0.2	0.2	2	0.964
0.2	0.5	1	0
0.2	0.5	2	0.975
0.2	1	1	0.002
0.2	1	2	0.972
⋮	⋮	⋮	⋮

$\lambda = [0 \ \ .2 \ \ .5 \ \ 1 \ \ 5]$

$\rho = [0 \ .1 \ \ .2 \ \ .5 \ \ 1]$

Cross Validation Results

λ	ρ	prediction error
0	0	0.803
0	0.2	0.799
0	0.5	0.795
0	1	0.791
0	5	0.796
0.1	0	0.801
0.1	0.2	0.798
0.1	0.5	0.794
0.1	1	0.790
0.1	5	0.797
0.2	0	0.799
0.2	0.2	0.796
0.2	0.5	0.793
0.2	1	0.789
0.2	5	0.798
0.5	0	0.795
0.5	0.2	0.793
0.5	0.5	0.790
0.5	1	0.788
0.5	5	0.801
1	0	0.790
1	0.2	0.789
1	0.5	0.788
1	1	0.787
1	5	0.807

$\lambda = [0\ \ .2\ \ .5\ \ 1\ \ 5]$

$\rho = [0\ .1\ \ .2\ \ .5\ \ 1]$

Comparison in estimating B

Non-Regularization

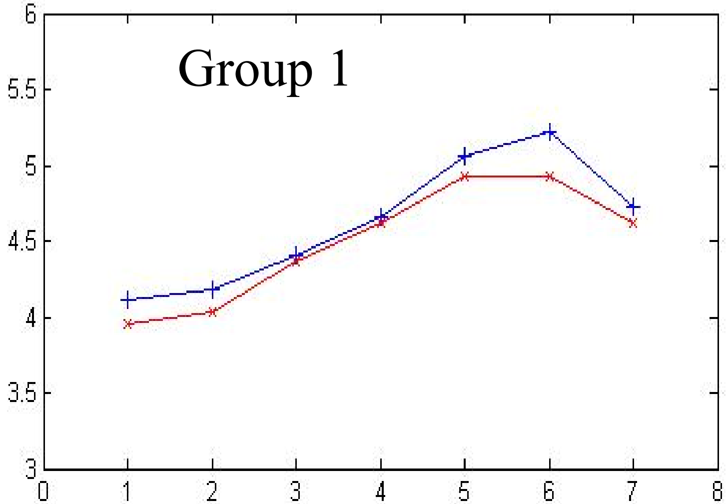
$(\lambda = 0 \;,\; \rho = 0)$

	Const.	Linear	Quad	Cub
g1	.566* (.148)	.195* (.060)	-.057 (.053)	-.094* (.038)
g2	-.498* (.163)	-.172* (.047)	.050 (.044)	.083* (.029)
g3	.116 (.198)	.040 (.066)	-.012 (.031)	-.020 (.029)
g4	-.116 (.161)	-.040 (.056)	.012 (.023)	.020 (.028)

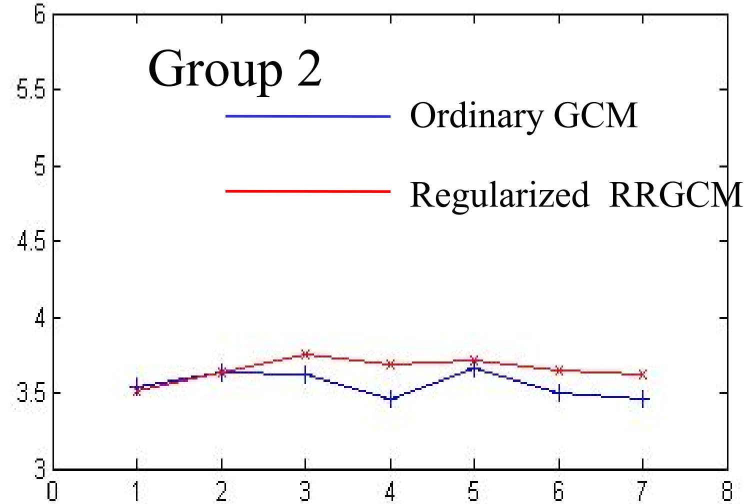
Regularization

$(\lambda = 1 \;,\; \rho = 1)$

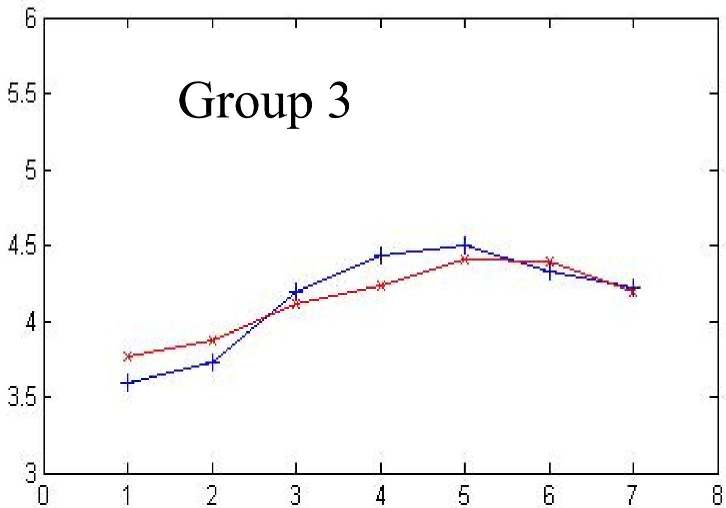
	Const.	Linear	Quad	Cub
g1	.446* (.117)	.154* (.047)	-.045 (.039)	-.074* (.029)
g2	-.396* (.124)	-.137* (.037)	.040 (.034)	.066* (.022)
g3	.090 (.158)	.031 (.053)	-.009 (.024)	-.015 (.023)
g4	-.092 (.124)	-.035 (.043)	.009 (.018)	.015 (.022)



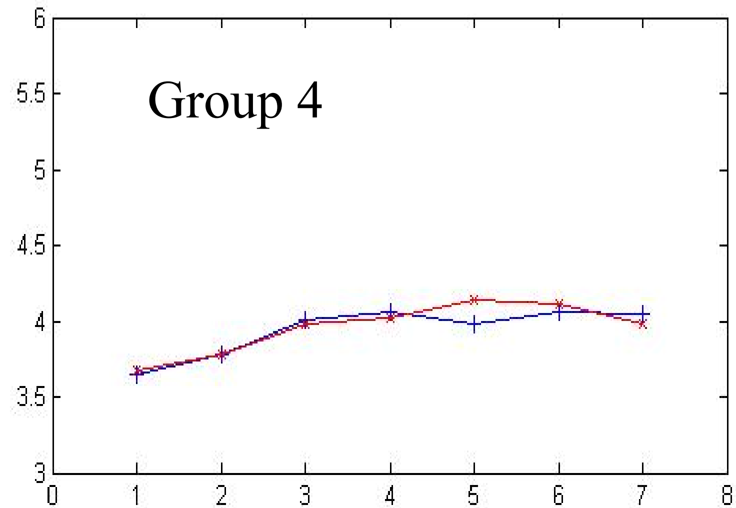
Control group



extrinsic cardiac denervation three weeks



extrinsic cardiac denervation immediately



thoracic sympathectomy three weeks

Conclusions

- Regularized reduced-rank LS estimate of B
  - Derivation
  - Usefulness
- Further extension
  - A mixture model of GCM and MANOVA for capturing time-invariant covariates

$$Y = X_1 B_1 H_1' + X_2 B_2 + E$$

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Thank you

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