Thresholding Nonparametric Regression

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Abstract

We propose a new thresholding approach in the area of nonparametric regression. Though following the Basis-based method, the new approach provides a nonlinear estimator by incorporating the feature of thresholding shrinkage. In an attempt to remove those significantly small coefficients measuring the detailed structure of the undelying curve while retain those large ones measuring the gross structure, we reconstruct the original model by an orthogonal transformation to detect those nonzero coefficients. The nonzero eigenvalue or singular values of the design matrix can be used as such indications for the smoothness of the underlying function is in proportion to the singular values. Consequently the use of singular value decomposition is proposed. This allows us to reduce high dimensionality and to find reasonable estimators. Then the minimization of the high-dimensional penalized least squares reduces to componentwise minimization problems and the solutions can be easily obtained with relevant thresholding rules and parameters as well. Therefore the selection of an optimal thresholding parameter and resulting thresholding rules is the crucial part of our method. A number of popular selection criteria such as (CV, GCV, AIC, BIC) have been introduced and the corresponding results examined in details. The appealing features of the application of SVD and componentwise penalized least-squares lies not only in the coefficientwise estimation that is computationally convenient, but also in the data dependent thresholding rule and the relevant local adaptive thresholding parameter. These data dependent thresholding rules show ideal properties in shrinking coefficients and retaining gross features with spatial adaption and computational convenience as well. An elaborated simulation study is carried out to support our claims.

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