

Information theoretic approaches to item selection

Alexander Weissman
Law School Admission Council
661 Penn Street, Newtown, PA 18940 USA
aweissman@lsac.org

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Abstract

A common criterion for item selection in a test is the information that an item or set of items provides. Information in this regard is most commonly defined as Fisher information. However, Fisher information is only one possible definition for the information content of an item or items. Information theory suggests a definition of information based on Shannon entropy (Cover and Thomas, 1991).

Information theoretic approaches to item selection have been considered mostly in the context of computerized adaptive testing (e.g., Chang & Ying, 1996; Eggen, 1999; Chen, Ankenmann & Chang, 2000; Cheng & Liou, 2000) by utilizing the Kullback-Leibler distance, or relative entropy (Cover and Thomas, 1991). Defined as the relative entropy between two probability distributions, it has been employed to select items based on the “global” information content of an item with respect to two values on the latent trait continuum. The form of Kullback-Leibler distance makes it particularly convenient for two-category adaptive classification tests (Eggen, 1999).

Nevertheless, utilizing Kullback-Leibler distance for more than two points is cumbersome. Rudner (2002) suggested a related approach to adaptive classification tests utilizing conditional entropy. It is shown here that this approach may be formulated in terms of mutual information. Mutual information possesses symmetry properties and may easily be extended to multiple categories for an adaptive classification test or to tests where an estimate of the continuous latent trait is desired. Results of simulation studies using mutual information will be reported.

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