# Do the Test Statistics for ML, NWLS and GLS Follow a Non-central Chi-square Distribution under Model Misspecification?

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## Abstract

Over the years several discrepancy functions have been introduced both in the literature and in the software of Structural Equation Modeling (SEM). The test statistics for discrepancy functions such as Maximum Likelihood (ML), Generalized Least Squares (GLS) and Normal Theory Weighted Least Squares (NWLS) are all asymptotically equivalent and approximate a central Chi-square statistic when the models are correctly specified and the observed variables are multivariate normally distributed. However, it is known that the distribution of these test statistics will not approximate a central Chi-square distribution for models containing specification error, but is more likely to follow a non-central Chi-square distribution (Browne 1984). This study investigates the empirical distributions of the test statistics for the three discrepancy functions, ML, GLS and NWLS, with regard to their ability to approximate a theoretical non-central Chi-square distribution, when models contain specification error.
The study includes different factor models with different types and degrees of specification error. Furthermore, the models also vary with respect to the number of included variables. Based on a design similar to that proposed by Cudeck and Browne (1992) we investigate the effect of different levels of specification error, represented by different levels of $F_0$ for the various conditions. $F_0$ is held constant at the same value for the three estimators. Consequently, if the stochastic variable $n\hat{F}$ is non-central Chi-square distributed, the test statistic of the three estimators should behave similarly with identical means and variances. However, we found that the empirical distributions of $n\hat{F}$ was not identical for the three estimators. Also, the non-central Chi-square approximation was found, in some cases, to be not appropriate even for models that contained minor and moderate degrees of specification error.

**References:**

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