A Model For Subjective Probability Data

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Abstract

In this presentation a new model for subjective probability data is proposed. Subjective probability data are obtained when subjects provide for each alternative of a multiple choice item their (subjective) probability that it is correct. One can either ask subjects to express their subjective probabilities, or ask them to distribute a number of "marbles" to the alternatives according to their subjective probabilities. A model will be proposed to relate the subjective probabilities to the underlying ability. This model is closely related to the Dirichlet-Multinomial model of Verstralen and Verhelst (2000). It differs from this model in so far that our model implies the logarithmic scoring rule of Shuford, et al. (1966) as a sufficient statistic for the ability of the subject.

The key assumption in our model is that the subjective probabilities can be conceived of as the choice probabilities of a forced choice experiment. These choice probabilities are modeled in accordance with Luce's Choice Axiom (Luce, 1959).

It will further be shown how subjective probability responses relate to other ways of responding to multiple-choice items. Specifically, we show that our model implies the three parameter logistic model (Birnbaum, 1969) if only a single marble is to be assigned and we only register whether it is assigned to the correct alternative or not.

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