Positive loadings and factor correlations for positive manifold data

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Abstract

"It is a universally accepted fact that intertest correlations for mental abilities are positive" (Thurstone, 1947, p. 342). A covariance matrix is said to have positive manifold if its elements are positive. Hence, if the covariance matrix has positive manifold for sample sizes sufficiently large, then the population covariance matrix has positive manifold. This assumption of positive manifold implies for the single factor analysis model that the loadings as well as the regression weights for best linear factor prediction are positive. This result pertains to the $g-$factor model, the model in classical test theory, and the normal ogive response model (Takane & De Leeuw, 1987).

For the multiple factor analysis model where each variable loads on a single factor and a hierarchical factor model (Yung, Thissen, & Cleoid, 1999), the assumption of positive manifold implies that the loadings, the factor correlations, and (somewhat loosly) the regression weights are positive. This pertians to models for sets of congeneric measurements (Jöreskog, 1971), models for factor analysis in several populations, and certain structural equation models. Positive regression weights imply that the true factor is near the positive orthant of the observable variables.

If the jacobian matrix corresponding to the parameter vector of the model has full column rank, the model are locally identified. Hence, the weak consistency of the sequence of parameter vectors generated by (generalized) least squares or maximum likelihood, implies that the estimated loadings, factor correlations, and regression weights are positive. The latter explains why in many applications positive loadings and factor correlations are encountered.

References

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