

Estimating polytomous item response functions with free-knot splines

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Abstract

One of the goals of item response theory (IRT) is to estimate the category response functions (CRF; $P_{jk}(\theta) = Pr\{X_j = k \mid \theta\}$). There are a number of parametric forms for the CRFs (e.g. partial credit model, graded response model, sequential model). These simple models often prove to be too restrictive for many data sets. This paper estimates the CRFs with cubic B-splines by: (a) making the necessary constraints on the spline coefficients to ensure $\sum_{k=0}^t P_{jk}(\theta)$ is monotone increasing in θ for all t ; and (b) treating the number and location of the knots as unknown parameters. Because we treat the number of knots as an unknown parameter, the parameter space is a disjoint union of spline spaces, and conventional estimation procedures are invalid. For this reason we use a reversible-jump Markov chain Monte Carlo algorithm to estimate the CRFs. We demonstrate the practicality of the spline model on a data set from behavioral psychology.

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