Fitting a vector model to the transition matrices

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Abstract
When we are interested in the market share of n objects, the main data matrix to be analyzed is the $n \times n$ Brand Switching matrix from time $t-1$ to time t. By analyzing this transition matrix, we can characterize the change of market share of n objects between two time points.
one approach to analyze is fitting a Markov chain model, or a probit model by assuming the stationary process. Other one is fitting the two-way two-mode MDS(MultiDimensional Scaling) method such as MDSCAL. These are useful. However, when there are more than two transition matrices, and it is indicated some eventual change occurred, these approach are not appropriate. Then one is applying multidimensional scaling methods to asymmetric similarity data, such as INDSCAL(Carroll and Chang, 1970), and two-mode three-way asymmetric MDS(Okada and Imaizumi 1997). This approach is useful, but is difficult to predict the future trend of Brand Switching of n objects. We can also apply the congruence matching methods such as Ten Berg proposed (1977) when we focus on the change of structure. In those models, it is difficult to explain some eventual change such as the change of the car market share induced by new car. And we propose new approach to explain the Brand Switching matrices of two-mode three-way. Let $B(t)(t = 1, 2, \cdots, T)$ be $n \times n$ transition matrix, each cell of $B(t)$ represent the transition frequency from row object o_i at time $t-1$ to column object o_j at time t. We assume the geometric representation of the market share of n objects, $X(t)$, and the event representation at time t by n dimensional vector $v(t)$, and also assume $X(t) = f((X(t-1), D(t), T(t)) + v(t)), with X(t-1)'v(t) = 0$. Here $D(t)$ is diagonal matrix, $T(t)$ is orthonormal matrix. By introducing the term $v(t)$, we try to explain the event which occurs at the time point t. We propose the model which relate $X(t)$ and $X(t-1)$ with $B(t)$ by $B_{ij}(t) = g(dist(X_i(t-1), X_j(t)))$, here $dist(x, y)$ is the distance between point x and y. The application to the real data set of the cars in Japan market will be shown.

References

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