Multidimensional Scaling in the City-Block Metric: L_1 and L_2-Norm Optimization Methods Using MATLAB

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Abstract

In a recent paper by Hubert, Arabie, and Meulman (2002), a comparison is made among several different optimization strategies for the linear unidimensional scaling (LUS) task in the L_2-norm, with all implementations carried out within a MATLAB computational environment. The central LUS task involves arranging the n objects in a set $S = \{O_1, O_2, \ldots, O_n\}$ along a single dimension, defined by coordinates $x_1, x_2, \ldots, x_n$, based on an $n \times n$ symmetric proximity matrix $\mathbf{P} = \{p_{ij}\}$, whose nonnegative entries are given a dissimilarity interpretation ($p_{ij} = p_{ji}$ for $1 \leq i, j \leq n$; $p_{ii} = 0$ for $1 \leq i \leq n$). The L_2 criterion

$$\sum_{i<j} (p_{ij} - |x_j - x_i|)^2, \tag{1}$$

is minimized by the choice of the coordinates. The present paper can be considered a close companion to this earlier piece, with extensions now given to multidimensional scaling in the city-block metric for both the L_1 and L_2 norms. The computational routines to be discussed and illustrated are again freely available as MATLAB m-files.

Although L_1 norms are possible to use within both the unidimensional and multidimensional contexts, and we give MATLAB m-files to do so, our general conclusion after experimentation is that they might be best avoided because of their generally needed much greater computationally expensive implementation without any particularly clear advantage; there is also the disconcerting periodic warnings of possible ill-conditioning for the repetitive linear programming subtasks when using the MATLAB Optimization Toolbox m-file `linprog.m`.

References

- Hubert, L. J., Arabie, R., & Meulman, J. J. (2002). Linear unidimensional scaling in the L_2-norm: Basic optimization methods using MATLAB. *Journal of Classification*, *19*, 303–328.