A Projection Pursuit Methodology for Blind Signal Extraction

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Abstract

The current theory for Independent Component Analysis (ICA) tries to model the observations as unknown linear combination or mixture of N independent components or sources $S_1(t), \ldots, S_N(t)$ whose distribution is also usually unknown. In the ICA problem one tries to recover all the N independent and non-Gaussian components from the only knowledge of the observations. In this paper, we address the generalization of the problem where one tries to estimate with $P \leq N$ outputs $Y_1, \ldots, Y_P$ (linear function of the observations) a subset of P of the N independent components. The paper presents, from an unified standpoint, several existing contrast for ICA (minimum mutual information, minimum entropy, maximum likelihood, negentropy, infomax, ...) but here considering the case of the extraction of a subset P of sources. This extension derives from the projection pursuit methodology presented in [1-4] and from the following related set of inequalities (where $h(\cdot)$ denotes differential entropy)

$$-\sum_{i=1}^{P} E[\log p_{\mathcal{S}_i}(y_i)] \geq \sum_{i=1}^{P} h(Y_i) \geq h(Y_1, \ldots, Y_P) \geq \sum_{i=1}^{P} h(S_i) \tag{1}$$

We also show how other semi-orthogonal contrasts or indexes $\psi(Y_i)$ for the extraction of a single independent component (like those based on cumulants) can be extended to the simultaneous extraction of P independent components, provided that $\psi(Y_i)$ verifies a certain convexity property with respect to the combination of the independent components. Finally, we present some practical algorithms that solve problem and discuss the similarities and differences between separation of all the components and the extraction of a subset of them.

References

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