

The Importance Of The Eigenvalues To Define A Configuration Of The Categories Of A Group Of Qualitative Variables

Colnago Efrem

Department Of Statistics, Probability, Applied Statistics
University Of Roma "La Sapienza", Piazzale Aldo Moro 5, Roma 00185 Italy
Efrem@Pow2.Sta.Uniroma1.It

Keywords: Categorical data, Burt's matrix, Quantification, Residual matrix, Linear and bilinear model

Abstract

There exist several methodologies for analyzing multivariate qualitative data (Scaling, Analysis of Correspondences, Dual scaling, Categorical Data, Multidimensional Scaling, and at last Data Mining). Our aim consists in finding a suitable constraint for discriminating and ordering the categories of the multivariate set of qualitative variables according to a unidimensional scale (categorical multidimensional data). Hence the problem: does an acceptable unidimensional scaling exist for this type of data and what constraints are needed to arrive at an optimal quantification of the categories? Nishisato (1993) defined in bivariate analysis, the minimum reliable level of the first eigenvalue to discriminate the categories. We will demonstrate, in our application in the multivariate analysis if this approach is reliable. He defines also two types of data matrix in quantifications. One is an "incidence data" matrix, where there exists a second component that influences the data of the first and another "dominance data" matrix, where this influence does not exist. Since we start from a Burt's matrix that usually is of the type "incidence data", then we want to verify with an example the influence of the second eigenvalue on the configuration of the first one. We introduce the PCA, as a model with linear and bilinear components (Gabriel 1971) and decompose the initial matrix into a sum of matrices related to the first eigenvalue and of residual traditional matrices. If we eliminate the contribution of the second and other successive eigenvalues we build a special residual matrix, that holds always the first factor. If we can show the independence of the configurations of the first component from the others in special residual matrices, we can demonstrate the non-influence of the second factor. So two methodological problems exist: the first eigenvalue can optimally order the categories (at what level?). Which is the acceptance interval of λ ?, based on the test $\chi^2(F)$. In this connection we introduce a non-negative matrix F formed by the configurations of the first eigenvalues in initial and successive special residual matrices. Besides we proceed, demonstrating the non-influence of the second component in traditional matrix with a Greenacre test and afterwards also in special residual matrices with a $\chi^2(F)^2$ on previous special multiway tables of configurations. We will prove, as (Guttman 1944) in scale opinions, (Nishisato 1993) in Dual Scaling, (Kop and Tournois 1995) in multidimensional scaling, when the information of the initial data matrix is sufficient to discriminate optimally the categories. (optimal quantification).

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