θ, but it solely depends on the item parameter a_g, when we use the normal ogive function for $P^*_{z_g}(\theta)$. From this result, we can easily see that both $I_V(\theta)$ and $I(\theta)$ are also constant, *i.e.*,

$$(7\text{-}5) \qquad I(\theta) = I_V(\theta) = \sum_{g=1}^{n} a_g^2 ,$$

provided that all the n items of the test have the normal ogive function as their $P^*_{z_g}(\theta)$.

In one of the previous works [Samejima, 1969, Chapter 6] the test information function $I(\theta)$ is illustrated for the normal ogive model by using six hypothetical items with the parameter value $a_g = 1/0.484$, $1/1.020$, $1/1.732$ respectively, for the three cases each in which an item is scored dichotomously, with three graded response categories, and with four graded response categories respectively. In these examples [cf. Samejima, 1969, p. 44, Figure 6–3] the highest value of $I(\theta)$ is less than 10 for $a_g = 1/0.484$, less than 3 for $a_g = 1/1.020$, and a little greater than 1 for $a_g = 1/1.732$. If we compute the values of $I(\theta)$ on the continuous response level by (7-5), the result is:

a_g	1/0.484	1/1.020	1/1.732
$I(\theta)$	25.61	5.77	2.00 .

We can see that all these values are much greater than those values obtained on the graded response level with rather small numbers of categories. This illustrates the fact that the continuous level situation is more informative than the graded level situation.

It has been pointed out [Samejima, 1969, Chapter 6] that the problem of *attenuation paradox* is ameliorated if we use graded items in place of dichotomous items, by illustrating with examples of hypothetical tests on the normal ogive model. It should be noted that the problem is completely solved if we use items of continuous responses, since the test information is constant for all θ, as is obvious from (7-5).

If we assume a multivariate normal distribution for the n *item variables* for some specified group of subjects, the item parameter a_g and the item response parameter b_{z_g} will be estimated as the direct expansion of the method for the dichotomous level [cf. Lord & Novick, 1968, Section 16.8]. In the present case, the observed value of z_g, which is denoted by $z_g^{(i)}$, where $i = 1, 2, \cdots, N$ and N is the sample size, is converted to the normal deviate, $\gamma_g^{(i)}$, in an appropriate way, and the product-moment correlation coefficient is computed on $\gamma_g^{(i)}$ for each pair of items. On this $n \times n$ correlation matrix, factor analysis is made, which is expected to give a single common factor. (If not, we must discard some items, and add some others if necessary, and apply factor analysis again, until clusters are eliminated.) Then the item parameter, a_g, is estimated from the common factor loading, in exactly the same way as in the dichotomous case [cf. Lord & Novick, 1968, page 375]. We divide $\gamma_g^{(i)}$ by the common factor loading, and denote the resulting statistic by $b_{z_g}^{(i)}$. It is obvious that $z_g^{(i)}$ and $b_{z_g}^{(i)}$ are strictly increasing in each other. We fit a curve for this relationship with an appropriate rationale which satisfies (3-1) and (4-8), and the resulting formula can be used for the conversion of *any* z_g into b_{z_g}.

(ii) Second, let us consider the logistic distribution function for $P_{z_g}^*(\theta)$, such that

$$P_{z_g}^*(\theta) = [1 + \exp\{-Da_g(\theta - b_{z_g})\}]^{-1}. \tag{7-6}$$

In this formula, D (>0) is a scaling factor, and 1.7 is often used for D, since it makes (7-6) very close to the normal ogive function and useful as its substitute, because of the convenient mathematical properties of the logistic function [Birnbaum, 1968, pages 399–401]. On the other hand, it has been pointed out [Samejima, 1969, pages 31–5; 1972, Sections 5.1, 5.2] that, in spite of the similarity of the curves given by their distribution functions, these two models have characteristically different properties when they are used as $P_{z_g}^*(\theta)$ on the graded response level.

Substituting (7-6) into (4-6), we have for the operating density characteristic

$$H_{z_g}(\theta) = Da_g P_{z_g}^*(\theta)[1 - P_{z_g}^*(\theta)]\left[\frac{d}{dz_g} b_{z_g}\right]. \tag{7-7}$$

The basic function is obtained by substituting (7-7) into (5-2) such that

$$A_{z_g}(\theta) = Da_g[1 - 2P_{z_g}^*(\theta)]. \tag{7-8}$$

Note this is a strictly decreasing and point-symmetric curve with $(b_{z_g}, 0)$ as the point of symmetry and Da_g and $-Da_g$ as its upper and lower asymptotes. This result is in contrast to that on the normal ogive model, for it is a straight line as we have just seen. The item score information function $I_{z_g}(\theta)$ is obtained by (6-2) and (7-8) such that

$$I_{z_g}(\theta) = 2D^2a_g^2P_{z_g}^*(\theta)[1 - P_{z_g}^*(\theta)], \tag{7-9}$$

which is a symmetric and uni-modal curve having its maximum at $\theta = b_{z_g}$. Figure 4 illustrates this curve for the case where $a_g = 1$ and $b_{z_g} = 0$, in contrast to the horizontal line obtained on the normal ogive model with the same value of the parameter a_g. Thus in the logistic model the accuracy of estimation shared by z_g in a response pattern is greatest at one point, *i.e.*, $\theta = 0$ in this example. This result also indicates that $I_V(\theta)$, the response pattern information function, is not constant for all θ, but depends on the n item scores, which consist of the response pattern V. The item information function $I_g(\theta)$ is given by substituting (7-9) into (6-3) such that